Solution-BTPES201T Engg. Maths-II

	P.E.S. COLLEGE OF ENGINEERING **(AN AUTONOMOUS INSTITUTE)** **CHH. SAMBHAJINAGAR- 431002** **Regular Summer Examination – 2026** **Course: F.Y.B. Tech.** **Branch: All** **Semester: II** **Subject Code & Name: BTPES201T Engineering Mathematics-II** **Max Marks: 60** **Date:29/06/2026** **Duration: 3 Hr.**		
	Instructions to the Students: *1. All the questions are compulsory.* *2. The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.* *3. Use of non-programmable scientific calculators is allowed.* *4. Assume suitable data wherever necessary and mention it clearly.*		
		(Level/CO)	Marks
Q. 1	**Solve Any six of the following.**		**6x2=12**
A)	$Find\ the\ modulus\ and\ argument(amplitude)of\ -1-i\sqrt{3}$ **Answer:** $modulus = r = 2$ -------------------1 mark $argument(amplitude) = \theta = \pi + \alpha = \pi + \frac{\pi}{3} = \frac{4\pi}{3}\ \ or\ 240^{\circ} --$1 mark	CO1	**2**
B)	$State\ the\ \ De - Moivre's theorem$ **Answer:**$If\ n\ is\ any\ real\ number\ then\ one\ of\ the\ values\ of$ $(cos\theta + isin\theta)^n = cosn\theta + isinn\theta ----$2 mark	CO1	**2**
C)	$Find\ the\ value\ of\ cosh(log2)$ **Answer**: $coshx = \frac{e^x + e^{-x}}{2} = \frac{e^{log2} + e^{-log2}}{2} --------$ 1 mark $= \frac{2 + \frac{1}{2}}{2} = \frac{5}{4} --------$ 1 mark	CO1	**2**
D)	$Solve\ \ (D^2 + 6D + 9)y = 0$ **Answer**: A. E. is$D^2 + 6D + 9 = 0\ \Rightarrow (D+3)^2 = 0$ $\therefore D = -3, -3\ --------$ 1 mark $\therefore y = (c_1 + c_2x)e^{-3x} --------$ 1 mark	CO2	**2**
E)	$Find\ the\ P.I.\ of\ the\ differential\ equation$ $(D^3 - 2D^2 - 5D + 6)y = e^{-x}$ **Answer**: $P.I. = \frac{1}{f(D)}X = \frac{1}{D^3 - 2D^2 - 5D + 6}e^{-x} ----$1 mark $= \frac{1}{8}e^{-x} ----$1 mark	CO2	**2**
F)	$If\ m_1, m_2\ are\ the\ real\ \&\ different\ roots\ of\ an\ auxillary\ \ equation$ $f(D) = 0, then\ what\ is\ the\ C.F.?$ **Answer**: $C.F. = c_1e^{m_1x} + c_2e^{m_2x} ----$2 mark	CO2	**2**

G)	$If\ f(x) is\ defined\ in\ the\ interval\ 0 < x < 2l,$ $then\ what\ is\ the\ value\ of\ a_n?$ **Answer**: $a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$ ----2 mark	CO3	**2**
H)	$If\ f(x) = sinx, 0 \le x \le 2\pi\ then\ find\ the\ value\ of\ a_0$ **Answer**: $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} sinx\,dx = \frac{1}{\pi}[-cosx]_{x=0}^{x=2\pi}$ --- 1 mark $= \frac{1}{\pi}[-cos2\pi + cos0] = 0$ --- 1 mark	CO3	**2**
I)	$If\ f(x) = x, -\pi \le x \le \pi\ then\ find\ the\ value\ of\ a_n$ **Answer**: $\boldsymbol{Given}\ \ f(x) = x, -\pi \le x \le \pi$ $put\ x = -x$ $\therefore f(-x) = -x = -f(x)$ --- 1 mark $\therefore f(x) = x\ is\ an\ odd\ function. \therefore\ a_n = 0$ --- 1 mark	CO3	**2**
Q.2	**Solve Any Two of the following.**		**12**
A)	$Prove\ that$ $(x+iy)^{m/n} + (x-iy)^{m/n} = 2(x^2+y^2)^{m/2n}\ cos\left[\frac{m}{n}tan^{-1}\left(\frac{y}{x}\right)\right]$ **Answer**: $Let\ x = rcos\theta\ and\ y = rsin\theta$ $where\ r = \sqrt{x^2+y^2}\ \therefore r = (x^2+y^2)^{1/2}\ and\ \theta = tan^{-1}\left(\frac{y}{x}\right)$ $\therefore x+iy = rcos\theta + irsin\theta = r(cos\theta + isin\theta) = re^{i\theta}\ and$ $x-iy = rcos\theta - irsin\theta = r(cos\theta - isin\theta) = re^{-i\theta}$ $Now, L.H.S. = (x+iy)^{m/n} + (x-iy)^{m/n} = \left(re^{i\theta}\right)^{m/n} + \left(re^{-i\theta}\right)^{m/n}$ $= r^{m/n}\left(e^{im\theta/n} + e^{-im\theta/n}\right)$ $using\ e^{i\theta} = cos\theta + isin\theta\ \ and\ \ e^{-i\theta} = cos\theta - isin\theta$ $\therefore L.H.S. = \left[(x^2+y^2)^{1/2}\right]^{m/n}\left[cos\left(\frac{im\theta}{n}\right) + isin\left(\frac{im\theta}{n}\right) + cos\left(\frac{im\theta}{n}\right)\right.$ $\left. - isin\left(\frac{im\theta}{n}\right)\right]$ $\therefore L.H.S. = (x^2+y^2)^{m/2n}\,2cos\left(\frac{im\theta}{n}\right) = 2(x^2+y^2)^{m/2n}\,cos\left[\frac{m}{n}tan^{-1}\left(\frac{y}{x}\right)\right]$ $= R.H.S.$	CO1	**6**
B)	$If\ cos(A+iB) = x+iy, prove\ that$ $(i)\frac{x^2}{cosh^2B} + \frac{y^2}{sinh^2B} = 1, (ii)\frac{x^2}{cos^2A} - \frac{y^2}{sin^2A} = 1$ **Answer**: Given $cos(A+iB) = x+iy$ $\Rightarrow cosAcosiB - sinAsiniB = x+iy$ $\Rightarrow cosAcoshB - isinAsinhB = x+iy$ $equating\ real\ and\ imaginary\ parts\ on\ both\ the\ sides, we\ have$	CO1	**6**

	$x = cosAcoshB - -①$ and $y = -sinAsinhB - -② - - -$ 2 mark $(i) L.H.S. = \frac{x^2}{cosh^2 B} + \frac{y^2}{sinh^2 B} = \frac{(cosAcoshB)^2}{cosh^2 B} + \frac{(-sinAsinhB)^2}{sinh^2 B}$ $= cos^2 A + sin^2 A = 1 = R.H.S. - - - -$ 2 mark $(ii) L.H.S. = \frac{x^2}{cos^2 A} - \frac{y^2}{sin^2 A} = \frac{(cosAcoshB)^2}{cos^2 A} - \frac{(-sinAsinhB)^2}{sin^2 A}$ $= cosh^2 B - sinh^2 B = 1 = R.H.S. - - - -$ 2 mark		
C)	*Find the area of the circle* $x^2 + y^2 = a^2$ ***Answer***: $Area = A = \iint dxdy = 4\int_{y=0}^{y=a}\int_{x=0}^{x=\sqrt{a^2-y^2}} dx\,dy - - -$ 3 mark $= \pi a^2 - -$3 mark $OR\ Area = A = \iint dxdy = \int_{x=0}^{x=a}\int_{y=0}^{y=\sqrt{a^2-x^2}} dy\,dx \quad = \pi a^2$	CO5	**6**
Q. 3	**Solve Any Two of the following.**		**12**
A)	$Solve (D^3 + D^2 + D + 1)y = sin2x$ ***Answer***: A. E. is$D^3 + D^2 + D + 1 = 0 \Rightarrow D = -1, \pm i \quad - - - -$1 mark $\therefore y = c_1 e^{-x} + e^{0x}(c_2 cosx + c_3 sinx) - - - - - - - -$ 1 mark $P.I. = \frac{1}{f(D)} X = \frac{1}{D^3 + D^2 + D + 1} sin2x = \frac{1}{-4D - 4 + D + 1} sin2x$ $= -\frac{1}{45}(3sin2x - 6cos2x) \ - - - -$3 mark *The general solution is* $y = C.F.. + P.I.$ $= c_1 e^{-x} + c_2 cosx + c_3 sinx - \frac{1}{45}(3sin2x - 6cos2x) - - -$ 1 mark	CO2	**6**
B)	*Solve by method of variation of parameters* $(D^2 + 6D + 9)y = \frac{e^{-3x}}{x^3}$ ***Answer***: A. E. is$D^2 + 6D + 9 = 0 \ \Rightarrow D = -3, -3$ $\therefore y = (c_1 + c_2 x)e^{-3x} = c_1 e^{-3x} + c_2 x e^{-3x} - - - - - -$ 1 mark $Let\ P.I. = Uy_1 + Vy_2 \ \ where\ y_1 = e^{-3x} \ , y_2 = xe^{-3x}$ $Wronskian = A = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & -3xe^{-3x} + e^{-3x} \end{vmatrix}$ $= e^{-6x} - - -$ 1 mark $U = -\int \frac{y_2}{A} X\ dx = -\int \frac{xe^{-3x}}{e^{-6x}} \frac{e^{-3x}}{x^3}\ dx = -\int \frac{1}{x^2}\ dx = \frac{1}{x} - -$1 mark $V = \int \frac{y_1}{A} X\ dx = \int \frac{e^{-3x}}{e^{-6x}} \frac{e^{-3x}}{x^3}\ dx = \int \frac{1}{x^3}\ dx = -\frac{1}{2x^2} - - -$ 1 mark $\therefore P.I. = Uy_1 + Vy_2 \ = \frac{e^{-3x}}{x} - \frac{xe^{-3x}}{2x^2} = \frac{e^{-3x}}{2x} - - - - - -$ 1 mark		

	$The\ general\ solution\ is$ $y = C.F..+P.I. = c_1e^{-3x} + c_2xe^{-3x} + \frac{e^{-3x}}{2x}$ − − − 1 mark		
C)	$Find\ the\ volume\ bounded\ by\ the\ cylinder\ x^2+y^2=4$ $and\ the\ planes\ y+z=3\ and\ z=0$ $\boldsymbol{Answer}: volume = \iiint dxdydz$ − − − 1 mark $= \int_{x=-2}^{x=2}\int_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}}\int_{z=0}^{z=3-y} dz\,dydx$ − − − 2 mark $= 12\pi$ − − − 3 mark	CO5	**6**
Q.4	**Solve Any Two of the following.**		**12**
A)	$Find\ the\ Fourier\ series\ of\ f(x) = \frac{1}{2}(\pi - x)\ over\ (0,2\pi)$ $\boldsymbol{Answer}: The\ Fourier\ series\ expansion\ of\ f(x)\ in\ the\ interval\ (0,2$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n cos(nx) + \sum_{n=1}^{n=\infty} b_n sin(nx)$ − − − − − − − − ① $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi}\frac{1}{2}(\pi - x)\,dx = \frac{1}{\pi}\left[\frac{\pi^2}{-2} - \frac{\pi^2}{-2}\right]$ $= 0$ − 1 mark $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\,cos(nx)dx = \frac{1}{\pi}\int_0^{2\pi}\frac{1}{2}(\pi - x)\,cos(nx)dx$ $integrating\ by\ parts\ using\ generalized\ rule$ $a_n = \frac{1}{2\pi}\left[(\pi - x)\left(\frac{sinnx}{n}\right) - (-1)\left(\frac{-cosnx}{n\times n}\right)\right]_{x=0}^{x=2\pi}$ $= \frac{1}{2\pi}\left\{\frac{(\pi - x)sinnx}{n} - \frac{cosnx}{n^2}\right\}_{x=0}^{x=2\pi}$ $\therefore a_n = \frac{1}{2\pi}\left[\frac{0}{n} - \frac{1}{n^2} - \left(\frac{0}{n} - \frac{1}{n^2}\right)\right] = \frac{1}{2\pi}\left[-\frac{1}{n^2} + \frac{1}{n^2}\right] = 0$ − − − 2 mark $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\,sin(nx)dx = \frac{1}{\pi}\int_0^{2\pi}\frac{1}{2}(\pi - x)\,sin(nx)dx$ $integrating\ by\ parts\ using\ generalized\ rule$ $b_n = \frac{1}{2\pi}\left[(\pi - x)\left(\frac{-cosnx}{n}\right) - (-1)\left(\frac{-sinnx}{n\times n}\right)\right]_{x=0}^{x=2\pi}$ $= \frac{1}{2\pi}\left\{\frac{-(\pi - x)cosnx}{n} - \frac{sinnx}{n^2}\right\}_{x=0}^{x=2\pi}$ $\therefore b_n = \frac{1}{2\pi}\left[\frac{\pi}{n} - \frac{0}{n^2} - \left(\frac{-\pi}{n} - \frac{0}{n^2}\right)\right] = \frac{1}{2\pi}\left[\frac{\pi}{n} + \frac{\pi}{n}\right] = \frac{1}{n}$ − − − 2 mark $put\ in\ equation$ ①$, we\ have$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n cos(nx) + \sum_{n=1}^{n=\infty} b_n sin(nx)$ $\therefore \frac{1}{2}(\pi - x) = \sum_{n=1}^{n=\infty}\frac{sin(nx)}{n}$ − − − 1 mark	CO3	**6**
B)	$Find\ the\ half\ range\ cosine\ series\ of\ f(x) = (x-1)^2$ $in\ the\ interval\ (0,1)$	CO3	**6**

	Answer: *The half range cosine series in the interval* $(0,l)$ *is given* $f(x)=\frac{a_0}{2}+\sum_{n=1}^{n=\infty} a_n cos\left(\frac{n\pi x}{l}\right)$ *compare the given interval* $(0,1)$*with the inteval* $(0,l)$ $\therefore l=1$ $\therefore f(x)=\frac{a_0}{2}+\sum_{n=1}^{n=\infty} a_n cosn\pi x ------- 1$ $a_0=\frac{2}{l}\int_0^l f(x)\,dx=\frac{2}{1}\int_0^1 (x-1)^2\,dx=2\left[\frac{(x-1)^3}{3}\right]_{x=0}^{x=1}$ $=2\left[0-\frac{(0-1)^3}{3}\right]=\frac{2}{3}---$ 2 mark $a_n=\frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx=\frac{2}{1}\int_0^1 (x-1)^2 cosn\pi x dx$ *integrating by parts(using generalised rule)* $\therefore a_n=2\left\{(x-1)^2\left(\frac{sinn\pi x}{n\pi}\right)-2(x-1)\left(\frac{-cosn\pi x}{n\pi\times n\pi}\right)\right.$ $\left.-2\left(\frac{-sinn\pi x}{n\pi\times n\pi\times n\pi}\right)\right\}_{x=0}^{x=1}$ $\therefore a_n=2\left\{\frac{(x-1)^2 sinn\pi x}{n\pi}+\frac{2(x-1)cosn\pi x}{n^2\pi^2}+\frac{2sinn\pi x}{n^3\pi^3}\right\}_{x=0}^{x=1}$ $\therefore a_n=2\left[-0-\left(0-\frac{2}{n^2\pi^2}+0\right)\right]=2\times\frac{2}{n^2\pi^2}=\frac{4}{n^2\pi^2}$ $\therefore a_n=\frac{4}{n^2\pi^2}---$ 3 mark *put in equation* 1 $\therefore f(x)=\frac{1}{3}+\frac{4}{\pi^2}\sum_{n=1}^{n=\infty}\frac{cosnx}{n^2}---$ 1 mark		
C)	*Find the surface area generated by revolving the loop of the curve* $9ay^2=x(3a-x)^2$ *about the* $x-axis$ *from* $x=0$ *to* $x=3a.$ ***Answer***: $S.A.=2\pi\int y\ ds$ $=2\pi\int_{x=0}^{x=3a} y\frac{ds}{dx}dx ----1 ---$ 1 mark *Given* $9ay^2=x(3a-x)^2\ \therefore\frac{dy}{dx}=\frac{(3a-x)(a-x)}{6ay}---$ 2 mark $Now,\frac{ds}{dx}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\frac{(3a-x)(a+x)}{6ay}---$ 2 mark *put in equation* 1 $\therefore S.A.=2\pi\int_{x=0}^{x=3a} y\frac{(3a-x)(a+x)}{6ay}dx=3a^2\pi\ ---$ 1 mark	CO5	**6**
Q. 5	**Solve Any Two of the following.**		**12**
A)	*Evaluate* $\iint xy\,dxdy$ *over the region bounded by* $x-axis, x=2a\ \&x^2=4ay$	CO4	**6**

	Answer: $Let\ I = \iint xy\,dxdy = \int_{y=0}^{y=a}\int_{x=\sqrt{4ay}}^{x=2a} xy\,dxdy$ $---$ 2 mark $= \int_{y=0}^{y=a}\left[\frac{x^2y}{2}\right]_{x=\sqrt{4ay}}^{x=2a} dy = \int_{y=0}^{y=a}\left[\frac{4a^2y}{2}-\frac{4ay^2}{2}\right]dy$ $= \left[\frac{2a^2y^2}{2}-\frac{4ay^3}{6}\right]_0^a = \frac{a^4}{3}$ $--$4 mark $OR\quad I = \iint xy\,dxdy = \int_{x=0}^{x=2a}\int_{y=0}^{y=x^2/4a} xy\,dydx = \frac{a^4}{3}$		
B)	$Change\ the\ order\ of\ integration\ \&\ evaluate\ \int_0^{\pi/2}\int_x^{\pi/2}\frac{cosy}{y}dxdy$ ***Answer***: $Let\ I = \int_0^{\pi/2}\int_x^{\pi/2}\frac{cosy}{y}dxdy$ $= \int_{y=0}^{y=\pi/2}\int_{x=0}^{x=y}\frac{cosy}{y}dxdy$ $-$ 3 mark $= 1$ $---$ 3 mark	CO4	**6**
C)	$Evaluate\ the\ integral\ I = \int_0^a\int_0^x\int_0^{x+y} dzdydx$ ***Answer***: $Let\ I = \int_0^a\int_0^x\int_0^{x+y} dzdydx = \int_{x=0}^{x=a}\int_{y=0}^{y=x}\int_{z=0}^{z=x+y} dzdydx$ $= \frac{a^3}{2}$ $----$6 mark	CO4	**6**
	***** End *****		

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