

“Please check whether you have got the right question paper.”

N.B

- i) Q No.1 and 6 are compulsory.
- ii) Solve any two questions from remaining of each section.
- iii) Figures to right indicate full marks.
- iv) Assume suitable data, if necessary.

SECTION-A

10

Q.1

Solve any five from the following

- a) If the real part of an analytic function $f(z) = u(r, \theta) + i\vartheta(r, \theta)$ is $-r \sin \theta$, then find $f(z)$.
- b) If $u = \cos ax \sin hy$ is harmonic, then find 'a'.
- c) Find the image of the line $y = -x$ under the transformation $W = z^2$.
- d) Evaluate $\int_0^{1+i} z^2 dz$ along $y = x$.
- e) Evaluate $\int_C \frac{1}{z-a} dz$, where C is the circle with center at 'a' and radius is r.
- f) Find the poles of the function and the corresponding residues at each pole of $f(z) = \frac{z+1}{z^2(z-2)}$
- g) Solve $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$

OR

Find the z-transform of $f(k) = \frac{1}{k}, k \geq 1$.

- h) Solve $\frac{\partial^2 z}{\partial x \partial y} = x$

OR

Find z-transform of $f(k) = e^{ka}, k \geq 0$

Q.2

- a) Test whether the function $f(x) = e^{-x}(\cos y - i \sin y)$ is analytic or not? 05
- b) Find the bilinear transformation which maps the points $z = 0, i, \infty$ onto the points $W = 0, \frac{1}{2}, \infty$. 05
- c) Solve $\frac{\partial^2 y}{\partial t^2} c^2 \frac{\partial^2 y}{\partial x^2}$, subject to the conditions $y(0, t) = 0, y(2l, t) = 0, \frac{\partial y}{\partial t} = 0$ at $t = 0$ and 05
 $y(x, 0) = \frac{bx}{l}, 0 \leq x \leq l$
 $\frac{b}{l}(2l - x), l \leq x \leq 2l$

OR

Find the z-transform of $\frac{2^k}{k}, k \geq 1$.

Q.3

- a) Prove that $u = r^3 \cos 3\theta - r \sin \theta$ harmonic, and find its conjugate harmonic. 05
- b) Evaluate $\int_0^{\pi - \pi i} e^{\bar{z}} dz$, along the curve $x=t, y=-t$. 05
- c) Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, with the conditions $u(1, t)$ for all $t \geq 0, \frac{\partial u}{\partial t} = 0$ at $x = 0$ and $u(x, 0) = 20x$ for $0 < x < 1$ 05

OR

Find the inverse z-transform of $\frac{z^3}{(z-3)(z-2)^2}, |z| > 3$

- Q.4 a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ for $0 < |z-1| < 2$. 05
- b) Evaluate $\oint_c \frac{\cos h iz}{z^2 + 4z + 3} dz$, where c is $|z|=2$ by Cauchy's integral formula. 05
- c) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions $u(0, y) = u(a, y) = 0$ for $0 \leq y \leq b$, $u(x, b) = 0$ and $u(x, 0) = x(a - x)$, for $0 \leq x \leq a$ 05

OR

Solve the difference equation by z-transform $Y_{n+2} + 2Y_{n+1} + Y_n = n$, $Y_0 = Y_1 = 0$

- Q.5 a) Show that the map of the real axis of z-plane on the W-plane under the transformation $W = \frac{1}{z+i}$ is a circle and find its radius and center. 05
- b) Evaluate $\oint_c \frac{dz}{z^2 \sin h z}$, where c is $|z-1|=2$ by Cauchy's Residue theorem. 05
- c) Evaluate $\int_0^{2\pi} \frac{1}{(5-3 \sin \theta)^2} d\theta$, by residues theorem. 05

SECTION -B

- Q.6 Solve any five from the following 10
- a) Find laplace transform of $\cos t \log t \delta(t - \pi)$
- b) Find $L[f(t)]$ and $L[f'(t)]$ of the function $f(t) = e^{-5t} \sin t$.
- c) Find Laplace transform of the function $f(t) = e^{t-a}, t > a$
 $0, t < a$
- d) Find the inverse Laplace transform of $\frac{4s+12}{s^2+8s+16}$
- e) Find the inverse Laplace transform of $\frac{s e^{-3s}}{s^2-1}$
- f) find inverse Laplace transform of $\frac{2+5s}{s^2}$
- g) Find the Fourier sine transform of $f(x) = k, 0 < x < a$
 $= 0, x < 0$.
- h) Find the Fournier transform of $f(x) = x e^{-x}, x > 0$
 $= 0, x < 0$.

- Q.7 a) Find the Laplace transform of $e^{-3t} \int_0^t t \sin 3t dt$. 05
- b) Find the inverse Laplace transform of $2 \tan h^{-1} s$. 05
- c) Using Fournier transform, solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, for $0 \leq x < \infty, t > 0$ given the conditions
i) $u(x, 0) = 0$ for $x \geq 0$ ii) $\frac{\partial u(0, t)}{\partial x} = -a$ (constant) iii) $u(x, t)$ is bounded 05

- Q.8 a) Evaluate $\int_0^\infty \frac{e^{-t} (1 - \cos 2t)}{2t} dt$ 05
- b) Find the inverse Laplace transform of $\frac{1}{(s-3)(s+3)^2}$ by convolution theorem. 05
- c) Solve the integral equation $\int_0^\infty f(x) \cos \lambda x dx = e^{-\lambda}, \lambda \geq 0$. 05

- Q.9 a) Express the following function in terms of Heaviside unit step function and hence find their Laplace transform
 $f(x) = t, 0 < t < 2$
 $t^2, t > 2$ 05
- b) Solve $y'' + 4y = \delta(t), y(0) = 0, y'(0) = 0$ by Laplace transform. 05
- c) Find the Fourier cosine transform of 05
 $f(x) = x, 0 < x < \frac{1}{2}$
 $= 1 - x, \frac{1}{2} < x < 1$
 $= 0, x > 1$
- Q.10 a) Find the Laplace transform of 05
 $f(t) = \cos wt, 0 < t < \pi/w$
 $0, \pi/w < t < 2\pi/w$ and $f(t) = f(t + \frac{2\pi}{w})$
- b) Find the Fourier transform of 05
 $f(x) = 1 + \frac{x}{a}, -a < x < 0$
 $= 1 - \frac{x}{a}, 0 < x < a$
 $= 0, \text{ otherwise}$
- c) Solve $\frac{dx}{dt} = y + e^t, \frac{dy}{dt} = \sin t - x, x(0) = 1, y(0) = 0$ by Laplace transform. 05